Permutations & Combinations

1. Eight chairs are numbered 1 to 8. Two women and 3 men wish to occupy one chair each. First the women choose the chairs from amongst the chairs 1 to 4 and then men select from the remaining chairs. Find the total number of possible arrangements.

   [Hint: 2 women occupy the chair, from 1 to 4 in ${}^4P_2$ ways and 3 men occupy the remaining chairs in ${}^6P_3$ ways.]

2. If the letters of the word RACHIT are arranged in all possible ways as listed in dictionary. Then what is the rank of the word RACHIT ?

   [Hint: In each case number of words beginning with A, C, H, I is 5!]

3. A candidate is required to answer 7 questions out of 12 questions, which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. Find the number of different ways of doing questions.

4. Out of 18 points in a plane, no three are in the same line except five points which are collinear. Find the number of lines that can be formed joining the point.

   [Hint: Number of straight lines = ${}^{18}C_2 - {}^5C_2 + 1$.]

5. We wish to select 6 persons from 8, but if the person A is chosen, then B must be chosen. In how many ways can selections be made?

6. How many committee of five persons with a chairperson can be selected from 12 persons.

   [Hint: Chairman can be selected in 12 ways and remaining in ${}^{11}C_4$.]

7. How many automobile license plates can be made if each plate contains two different letters followed by three different digits?

8. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected from the lot.

9. Find the number of permutations of n distinct things taken r together, in which 3 particular things must occur together.

10. Find the number of different words that can be formed from the letters of the word 'TRIANGLE' so that no vowels are together.

11. Find the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digit is to be repeated.

12. There are 10 persons named $P_1, P_2, P_3, \ldots P_{10}$. Out of 10 persons, 5 persons are to be arranged in a line such that in each arrangement P_1 must occur whereas P_4 and P_5 do not occur. Find the number of such possible arrangements.

    [Hint: Required number of arrangement = ${}^7C_4 \times 5!$]

13. There are 10 lamps in a hall. Each one of them can be switched on independently. Find the number of ways in which the hall can be illuminated.

[Hint: Required number = $2^{10} - 1$].

14. A box contains two white, three black and four red balls. In how many ways can three balls be drawn from the box, if atleast one black ball is to be included in the draw.

    [Hint: Required number of ways = ${}^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C_2 + {}^3C_3$.]

15. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then find ${}^rC_2$.

    [Hint: Form equation using $\frac{{}^nC_r}{{}^nC_{r+1}}$ and $\frac{{}^nC_r}{{}^nC_{r-1}}$ to find the value of r.]

16. Find the number of integers greater than 7000 that can be formed with the digits 3, 5, 7, 8 and 9 where no digits are repeated.

    [Hint: Besides 4 digit integers greater than 7000, five digit integers are always greater than 7000.]

17. If 20 lines are drawn in a plane such that no two of them are parallel and no three are concurrent, in how many points will they intersect each other?

18. In a certain city, all telephone numbers have six digits, the first two digits always being 41 or 42 or 46 or 62 or 64. How many telephone numbers have all six digits distinct?

19. In an examination, a student has to answer 4 questions out of 5 questions; questions 1 and 2 are however compulsory. Determine the number of ways in which the student can make the choice.

20. A convex polygon has 44 diagonals. Find the number of its sides.

    [Hint: Polygon of n sides has $({}^nC_2 - n)$ number of diagonals.]

Long Type Questions

21. 18 mice were placed in two experimental groups and one control group, with all groups equally large. In how many ways can the mice be placed into three groups?

22. A bag contains six white marbles and five red marbles. Find the number of ways in which four marbles can be drawn from the bag if (a) they can be of any colour (b) two must be white and two red and (c) they must all be of the same colour.

23. In how many ways can a football team of 11 players be selected from 16 players? How many of them will

(i) include 2 particular players?

(ii) exclude 2 particular players?

24. A sports team of 11 students is to be constituted, choosing at least 5 from Class XI and atleast 5 from Class XII. If there are 20 students in each of these classes, in how many ways can the team be constituted?

25. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has

(i) no girls

(ii) at least one boy and one girl

(iii) at least three girls.

Objective Type Questions

26. If ${}^{n}C_{12} = {}^{n}C_{8}$, then n is equal to

(A) 20 (B) 12 (C) 6 (D) 30

27. The number of possible outcomes when a coin is tossed 6 times is

(A) 36 (B) 64 (C) 12 (D) 32

28. The number of different four digit numbers that can be formed with the digits 2, 3, 4, 7 and using each digit only once is

(A) 120 (B) 96 (C) 24 (D) 100

29. The sum of the digits in unit place of all the numbers formed with the help of 3, 4, 5 and 6 taken all at a time is

(A) 432 (B) 108 (C) 36 (D) 18

30. Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to

(A) 60 (B) 120 (C) 7200 (D) 720

31. A five digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4 and 5 without repetitions. The total number of ways this can be done is

(A) 216 (B) 600 (C) 240 (D) 3125

[Hint: 5 digit numbers can be formed using digits 0, 1, 2, 4, 5 or by using digits 1, 2, 3, 4, 5 since sum of digits in these cases is divisible by 3.]

32. Every body in a room shakes hands with everybody else. The total number of hand shakes is 66. The total number of persons in the room is

(A) 11 (B) 12 (C) 13 (D) 14

33. The number of triangles that are formed by choosing the vertices from a set of 12 points, seven of which lie on the same line is

(A) 105 (B) 15 (C) 175 (D) 185

34. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is

(A) 6 (B) 18 (C) 12 (D) 9

35. The number of ways in which a team of eleven players can be selected from 22 players always including 2 of them and excluding 4 of them is

(A) ${}^{16}C_{11}$ (B) ${}^{16}C_{5}$ (C) ${}^{16}C_{9}$ (D) ${}^{20}C_{9}$

36. The number of 5-digit telephone numbers having atleast one of their digits repeated is

(A) 90,000 (B) 10,000 (C) 30,240 (D) 69,760

37. The number of ways in which we can choose a committee from four men and six women so that the committee includes at least two men and exactly twice as many women as men is

(A) 94 (B) 126 (C) 128 (D) None

38. The total number of 9 digit numbers which have all different digits is

(A) 10! (B) 9 ! (C) $9 \times 9!$ (D) $10 \times 10!$

39. The number of words which can be formed out of the letters of the word ARTICLE, so that vowels occupy the even place is

(A) 1440 (B) 144

(C) 7! (D) ${}^4C_4 \times {}^3C_3$

40. Given 5 different green dyes, four different blue dyes and three different red dyes, the number of combinations of dyes which can be chosen taking at least one green and one blue dye is

(A) 3600 (B) 3720 (C) 3800 (D) 3600

[Hint: Possible numbers of choosing or not choosing 5 green dyes, 4 blue dyes and 3 red dyes are 2^5, 2^4 and 2^3, respectively.]

Fill in the Blanks Type Questions

41. If ${}^nP_r = 840$, ${}^nC_r = 35$, then $r =$ _____.

42. ${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7 =$ _____.

43. The number of permutations of n different objects, taken r at a line, when repetitions are allowed, is

44. The number of different words that can be formed from the letters of the word INTERMEDIATE such that two vowels never come together is _____.

[Hint: Number of ways of arranging 6 consonants of which two are alike is $\frac{6!}{2!}$ and number of ways of arranging vowels $= {}^7P_6 \times \frac{1}{3!} \times \frac{1}{2!}$.]

45. Three balls are drawn from a bag containing 5 red, 4 white and 3 black balls. The number of ways in which this can be done if at least 2 are red is _____

46. The number of six-digit numbers, all digits of which are odd is _____.

47. In a football championship, 153 matches were played . Every two teams played one match with each other. The number of teams, participating in the championship is _____.

48. The total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two signs '–' occur together is _____.

49. A committee of 6 is to be chosen from 10 men and 7 women so as to contain atleast 3 men and 2 women. In how many different ways can this be done if two particular women refuse to serve on the same committee.

[Hint: At least 3 men and 2 women: The number of ways = ${}^{10}C_3 \times {}^{7}C_3 + {}^{10}C_4 \times {}^{7}C_2$. For 2 particular women to be always there: the number of ways = ${}^{10}C_4 + {}^{10}C_3 \times {}^{5}C_1$. The total number of committees when two particular women are never together = Total – together.]

50. A box contains 2 white balls, 3 black balls and 4 red balls. The number of ways three balls be drawn from the box if at least one black ball is to be included in the draw is _____.

True or False Type Questions

51. There are 12 points in a plane of which 5 points are collinear, then the number of lines obtained by joining these points in pairs is ${}^{12}C_2 - {}^{5}C_2$.

52. Three letters can be posted in five letterboxes in 3^5 ways.

53. In the permutations of n things, r taken together, the number of permutations in which m particular things occur together is ${}^{n-m}P_{r-m} \times {}^{r}P_{m}$.

54. In a steamer there are stalls for 12 animals, and there are horses, cows and calves (not less than 12 each) ready to be shipped. They can be loaded in 3^{12} ways.

55. If some or all of n objects are taken at a time, the number of combinations is $2^n - 1$.

56. There will be only 24 selections containing at least one red ball out of a bag containing 4 red and 5 black balls. It is being given that the balls of the same colour are identical.

57. Eighteen guests are to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on other side of the table. The number of ways in which the seating arrangements

can be made is $\frac{11!}{5!6!}(9!)(9!)$.

[Hint: After sending 4 on one side and 3 on the other side, we have to select out of 11; 5 on one side and 6 on the other. Now there are 9 on each side of the long table and each can be arranged in 9! ways.]

58. A candidate is required to answer 7 questions out of 12 questions which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. He can choose the seven questions in 650 ways.

59. To fill 12 vacancies there are 25 candidates of which 5 are from scheduled castes. If 3 of the vacancies are reserved for scheduled caste candidates while the rest are open to all, the number of ways in which the selection can be made is ${}^{5}C_{3} \times {}^{20}C_{9}$.

Match Type Questions

In each if the Exercises from 60 to 64 match each item given under the column C_1 to its correct answer given under the column C_2.

60. There are 3 books on Mathematics, 4 on Physics and 5 on English. How many different collections can be made such that each collection consists of :

C_1	C_2
(a) One book of each subject;	(i) 3968
(b) At least one book of each subject :	(ii) 60
(c) At least one book of English:	(iii) 3255

61. Five boys and five girls form a line. Find the number of ways of making the seating arrangement under the following condition:

C_1	C_2
(a) Boys and girls alternate:	(i) $5! \times 6!$
(b) No two girls sit together :	(ii) $10! - 5!6!$
(c) All the girls sit together	(iii) $(5!)^2 + (5!)^2$
(d) All the girls are never together :	(iv) $2!5!5!$

62. There are 10 professors and 20 lecturers out of whom a committee of 2 professors and 3 lecturer is to be formed. Find :

	C_1	C_2
(a)	In how many ways committee : can be formed	(i) $^{10}C_2 \times {}^{19}C_3$
(b)	In how many ways a particular : professor is included	(ii) $^{10}C_2 \times {}^{19}C_2$
(c)	In how many ways a particular : lecturer is included	(iii) $^{9}C_1 \times {}^{20}C_3$
(d)	In how many ways a particular : lecturer is excluded	(iv) $^{10}C_2 \times {}^{20}C_3$

63. Using the digits 1, 2, 3, 4, 5, 6, 7, a number of 4 different digits is formed. Find

	C_1	C_2
(a)	how many numbers are formed?	(i) 840
(b)	how many numbers are exactly divisible by 2?	(ii) 200
(c)	how many numbers are exactly divisible by 25?	(iii) 360
(d)	how many of these are exactly divisble by 4?	(iv) 40

64. How many words (with or without dictionary meaning) can be made from the letters of the word MONDAY, assuming that no letter is repeated, if

	C_1	C_2
(a)	4 letters are used at a time	(i) 720
(b)	All letters are used at a time	(ii) 240
(c)	All letters are used but the first is a vowel	(iii) 360